

Emergence of small numbers in complex systems and the origin of the electroweak scale

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In sufficiently complex models with many parameters that are unknown or undetermined from first principles, a small coupling or mass can naturally arise even if it is not protected by a symmetry or a result of some dynamics. On the example of the minimal supersymmetric model we demonstrate that, contrary to commonly accepted views, the electroweak scale up to 3 orders of magnitude below superpartner masses naturally occurs, without fine tuning of model parameters, and it is not probabilistically disfavored compared to any other possibility.

Introduction. In particle physics, a parameter, written in units of maximal energy at which a given model is valid, can naturally be significantly smaller than unity if it is protected by a symmetry from quantum corrections due to various interactions [1, 2]. When applied to the Higgs boson mass, which is related to the electroweak (EW) scale, this immediately implies the existence of new physics not far above the EW scale, since there is no symmetry in the standard model (SM) protecting the Higgs mass from radiative corrections [3–6]. The naturalness problem related to the Higgs boson mass is one of the most studied problems in particle physics and the main motivation for physics beyond the standard model.

Realizing that supersymmetry (SUSY) can provide the needed symmetry for any scalar mass [7], together with the intriguing picture of unification of strong and electroweak forces [8] and the possibility to build simple realistic models [9] made supersymmetric extensions of the SM the most promising candidates for new physics. In these models, the EW scale is a result of quantum corrections and is calculable. These quantum corrections are comparable to masses of superpartners of SM particles and thus a generic prediction of these models, based on the naturalness argument, is that superpartners are at or very near the EW scale [10].

So far negative results of searches for superpartners, with some limits exceeding an order of magnitude above the EW scale, cast a shadow on this framework (similar concerns apply to other proposal for physics beyond the SM). Moreover, the measured value of the Higgs boson mass, taking aside naturalness considerations, indirectly points to superpartners two orders of magnitude above the EW scale [11]. Since the relevant parameters are masses squared, this would mean that the EW scale corresponds to a number which is 4 orders of magnitude smaller than expected. It is commonly argued that such an outcome, apparently contradicting naturalness principle, would require fine tuning of model parameters at the level of 1 part in 10^4 , which is unlikely to happen without an additional symmetry or dynamical mechanism [10].

In this letter we argue that in sufficiently complex mod-

els with many parameters that are unknown or undetermined from first principles, a small coupling or mass can naturally arise even if it is not protected by a symmetry or a result of some dynamics. We will demonstrate that, when specific characteristics and complexity of supersymmetric extensions of the SM are taken into account, the electroweak scale up to 3 orders of magnitude below superpartner masses naturally occurs, without fine tuning of model parameters, and it is not probabilistically disfavored compared to any other possibility.

Besides complexity, the important characteristic of the EW symmetry breaking in supersymmetric models is that the EW scale is a result of cancellation of comparable contributions. In a toy model that closely mimics features of electroweak symmetry breaking, we demonstrate that in such situations, even if all the model parameters are specified with just one significant figure, the outcomes several orders of magnitude smaller compared to dominant contributions naturally arise with probabilities comparable to the most likely outcome.

Toy model. Let us consider an observable X , which is, up to much smaller contributions from other parameters in the model, given by the difference of two parameters A and B of comparable size:

$$X \simeq A - B. \quad (1)$$

We can always choose appropriate units so that A and B are random real numbers close to 1 and, in order to be specific, let us allow them to vary by 50% in both directions, between 0.5 and 1.5. The distribution of X is then an almost symmetric triangle peaked at 0, similar to that in Fig. 1(a) (which is slightly shifted to the right due to perturbations). For unequal intervals of A and B or their central values the distribution could be less peaked, more broad and shifted to the left or right. Nevertheless, as far as there is a non-negligible overlap in intervals over which A and B are allowed to vary, an outcome in the vicinity of zero will have a comparable probability to the most probable outcome.

If A and B were integers in overlapping ranges, there

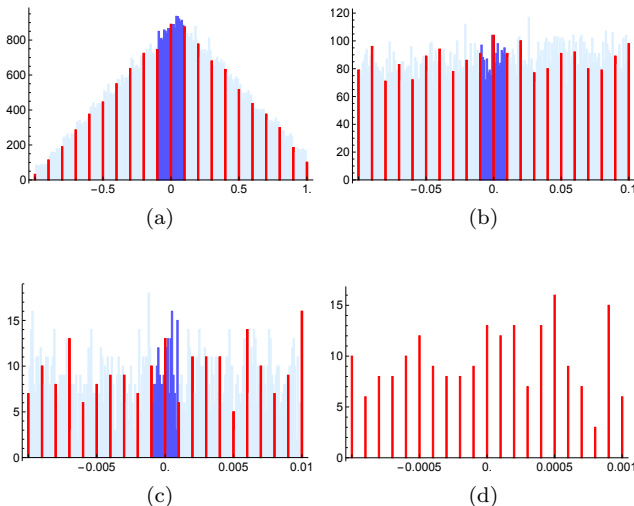


FIG. 1: Distribution of $X = R[1] - R[1] + R[0.1] + R[0.01] + R[0.001]$, where $R[x]$ is a randomly generated number from interval $[0.5x, 1.5x]$ keeping only the first significant figure specifying the departure from x . For example, $R[0.1]$ randomly generates numbers $0.1 + \{0, \pm 0.01, \pm 0.02, \dots, \pm 0.05\}$. The bin size of histograms is 0.01 (a), 0.001 (b), 0.0001 (c) and 0.00001 (d). The highlighted region near zero in plot (a) is shown in plot (b) and so on. The highlighted lines (red) are the outcomes of $R[1] - R[1]$ (a), $R[1] - R[1] + R[0.1]$ (b), $R[1] - R[1] + R[0.1] + R[0.01]$ (c) and the complete X in (d).

would be no problem associated with X being a small number. It would be just one out of many possible comparably likely outcomes. Being integers supplies a natural bin size in which the outcomes are plotted and compared. For real numbers, an arbitrarily small outcome of X is possible. However it would require specifying A and B with many significant figures and having almost identical values. This, without any symmetry or dynamical reason, would seem like a huge coincidence and it is the essence of the fine tuning problem.

For example, if we specify the departure of A and B from 1 by one significant figure, the outcomes of X will be distributed in intervals of 0.1, indicated by highlighted lines in Fig. 1(a). A mismatch of the central values of A and B would shift the 0 outcome, but since the outcomes appear in 0.1 intervals, an X as small as 0.1 with either sign will naturally appear with probability comparable to the most likely outcome. A significantly smaller X would however require specifying departures of A and B from 1 by more than one significant figure.

However, in many physical systems there are more than two parameters contributing to a given observable. Let us consider a parameter C contributing to X at an order of magnitude smaller level than A and B . The exact value of C is not important since it only shifts the $A - B$ distribution slightly to the left or right and thus we can simply assume it is equal to 0.1. Let us again allow the parameter C vary in the interval ± 0.05 around the

central value, similarly to what we allowed for parameters A and B , and specify the departure from the central value with one significant figure. Adding C is then, up to a constant shift, equivalent to adding random numbers: $0.1 + \{\pm 0.01, \pm 0.02, \dots, \pm 0.05\}$. Let us define such a set of random numbers as $R[0.1]$. In such a notation, our A and B specified with one significant figure are $R[1]$. Varying C in steps corresponding to specifying it with one significant figure results in emergence of new outcomes of X in intervals of 0.01, or $0.1C$. These new outcomes are indicated by highlighted lines in Fig. 1(b).

If there are more parameters contributing at smaller levels, even if specified by just one significant figure, new outcomes will appear in intervals $\simeq 0.1$ of the corresponding parameter. The distribution of X resulting from adding three such perturbations, $R[0.1] + R[0.01] + R[0.001]$, is shown in different intervals in Fig. 1(a)-(d). In this specific example, outcomes as small as 10^{-4} appear and they are still about 10 times more likely than outcomes of order 1. No parameter needed to be specified with more than one significant figure, no tuning was required.

In order to obtain, without fine tuning, outcomes of X orders of magnitude smaller than contributions from A or B , it is necessary that a given model is sufficiently complex with at least several more parameters contributing to it. More importantly, contributions of these parameters have to be distributed in a way that the space of possible outcomes is continuously covered without any gaps. If, for example, there was a new parameter contributing to X at 10^{-8} level, it would not change our previous conclusions. This new contribution would only split the existing outcomes into new outcomes tightly clustered around those shown in Fig. 1(d). Because of the arbitrary shift from all the parameters of the model, the naturally occurring outcomes would still not be smaller than 10^{-4} .

It should be noted, that there is some degree of arbitrariness in deciding the ranges over which parameters are allowed to vary and the way their variations are parameterized. Different choices from the example above would result in somewhat different conclusions. Nevertheless, as far as the range is not negligible compared to the central value and the variation is specified by a number with one significant figure, a given parameter will result in the emergence of outcomes in intervals of 10% - 100% of its value. The final result and conclusions would not be modified by more than one order of magnitude.

Armed with the intuition from this toy model we can demonstrate that there is no problem associated with the electroweak scale being significantly below the scale of superpartners in supersymmetric extensions of the standard model. As in our toy model, the electroweak scale is a result of a difference of comparable numbers. In addition, the complexity of these models is enormous, with more than 100 parameters contributing to the determination of the EW scale.

EW symmetry breaking. In the standard model, EW symmetry breaking is the result of a negative quadratic term and a positive quartic term in the Higgs potential. The minimum of the potential, corresponding to the vacuum expectation value of the Higgs field, is away from zero and determines the scale of electroweak interactions and masses of Z , W and Higgs bosons.

One of the most attractive features of supersymmetric extensions of the SM is the fact that the negative mass squared term, triggering EW symmetry breaking, is obtained by radiative corrections in vast ranges of parameters of various models. The relevant mass squared parameter is a combination of the supersymmetric Higgs mass parameter, μ , and the soft SUSY breaking mass squared parameter, $m_{H_u}^2$,

$$|\mu|^2 + m_{H_u,0}^2 + \Delta m_{H_u}^2, \quad (2)$$

where we intentionally split the $m_{H_u}^2$ into the value generated by a SUSY breaking scenario at a given scale, $m_{H_u,0}^2$, and the radiative correction from the renormalization group (RG) running to the electroweak scale, $\Delta m_{H_u}^2$. The $|\mu|^2$ part could also be split in a similar way, but radiative corrections to this term are not dramatic and thus we use the EW scale value directly. The negative of the combination of parameters in Eq. (2) is directly related to the Z boson mass or the Higgs mass and thus it should be close to $(100 \text{ GeV})^2$ [10].

The power of radiative corrections is illustrated in Fig. 2 (a)–(c) where the RG evolution of m_{H_u} is shown together with the most contributing other parameters, the stop masses and the gluino mass for three representative cases. Although all superpartner masses can be free parameters, simple scenarios with just few parameters are commonly assumed. Here we assume a universal scalar mass, m_0 , a universal gaugino mass, $M_{1/2}$, and consider three cases: $M_{1/2} = 0$, $m_0 = 0$ and $M_{1/2} = m_0$. We also define $M_{SUSY} = \max(M_{1/2}, m_0)$ and the RG evolution in Fig. 2 is plotted in this unit starting at the grand unification scale (it is another simple and common assumption although it is not necessary for our discussion). In all three cases, and thus for any case in between, the $m_{H_u}^2$ parameter turns negative.

Remarkable feature of the RG flow is that $\Delta m_{H_u}^2$ eventually reaches $-M_{SUSY}^2$ but it doesn't run to much larger negative values. More specifically, for the case of m_0 domination, the $\Delta m_{H_u}^2 \simeq -M_{SUSY}^2$ after about 12 orders of magnitude of the RG flow. Similarly, in the case of gaugino domination, the same situation is achieved after 11 orders of magnitude of the RG flow and finally for $M_{1/2} = m_0$ we need about 9 orders of magnitude. In further RG evolution $\Delta m_{H_u}^2$ does not exceed $-1.2M_{SUSY}^2$, $-3M_{SUSY}^2$, and $-4M_{SUSY}^2$ respectively, even if evolved all the way to the EW scale which can be seen in Fig. 2 (d). Note, however, that stop masses no longer contribute to $\Delta m_{H_u}^2$ at energies below their masses. For stop masses

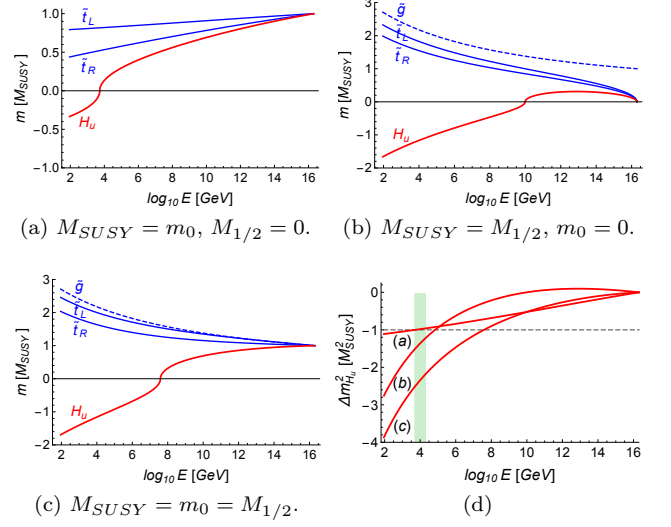


FIG. 2: RG evolution of the most relevant parameters contributing to the electroweak symmetry breaking. In (a), (b) and (c) the mass parameters of the gluino, \tilde{g} , left and right stops, $\tilde{t}_{L,R}$, and H_u , plotted as $m_{H_u}^2/|m_{H_u}^2|^{1/2}$ to account for $m_{H_u}^2$ turning negative at low energies, are plotted for three different boundary conditions in units of M_{SUSY} . In (d), the radiative correction, $\Delta m_{H_u}^2$, is plotted for the three cases, (a)–(c). The shaded region, 5 – 20 TeV, represents the range of stop masses which can result in the measured value of the Higgs boson mass without significant additional contributions.

which can result in the measured value of the Higgs boson mass without significant additional contributions, 5 – 20 TeV, indicated by shaded region in Fig. 2 (d), the $\Delta m_{H_u}^2$ ranges between $-1M_{SUSY}^2$ and $-2.5M_{SUSY}^2$.

The value of the μ term in Eq. (2) or its origin, although subject to intense research and usually the center of attention in naturalness considerations, is not crucial for our discussion. We assume $\mu \lesssim M_{SUSY}$ and sufficiently small not to prevent electroweak symmetry breaking. With $|\mu|^2 + m_{H_u,0}^2 \simeq M_{SUSY}^2$, either because $m_{H_u,0} \simeq M_{SUSY}$ or $\mu \simeq M_{SUSY}$ or both, we can see that the negative of the mass squared combination in Eq. (2) that sets the EW scale squared, M_{EW}^2 , is the result of cancellation of two comparable contributions as in our toy model: $X = -M_{EW}^2$, $A = |\mu|^2 + m_{H_u,0}^2 \simeq M_{SUSY}^2$ and $B = -\Delta m_{H_u}^2 \simeq M_{SUSY}^2$.

Although written in a simple way, the mass squared parameter in Eq. (2) depends on every single soft SUSY breaking scalar and gaugino mass, every gauge coupling and every Yukawa coupling in the model because they all contribute in the RG evolution to $\Delta m_{H_u}^2$. Assuming all soft masses comparable (not necessarily equal) of order M_{SUSY} at the grand unification scale, from two loop RG equations [12] we find that the approximate individual contributions to $\Delta m_{H_u}^2$ in units of M_{SUSY}^2 are as follows: -2.6 from gluino, -0.3 from each scalar top and H_u ,

0.2 from the $SU(2)$ gaugino, 0.05 from the $U(1)_Y$ gaugino, -4×10^{-3} from the trace of masses squared of all $SU(2)$ doublet scalars, -4×10^{-4} from the trace of masses squared of all scalars carrying hypercharge, $10^{-3} - 10^{-5}$ from each scalar bottom and H_d (for the ratio of vacuum expectation values of two Higgs doublets, $\tan \beta$, in the range $\tan \beta = 50 - 5$), and -4×10^{-6} from each scalar charm. After scalar charm contributions, there is a gap of about 5 orders of magnitude. Each scalar up contributes at 3×10^{-11} and the contribution from each scalar strange is about 10^{-8} of the contribution from scalar bottom.

In the list above, we did not include contributions from neutrino Yukawa couplings and scalar neutrinos, since these couplings are not known and are highly model dependent, and thus also from scalar charged leptons since contributions of these depend on the neutrino sector. In addition, we omitted the hypercharge weighted trace of all scalar masses squared which does not contribute if scalar masses are equal. Furthermore, we assumed diagonal Yukawa and scalar mass matrices, and neglected contributions from soft trilinear couplings that, if flavor diagonal, would contribute at comparable levels to scalar masses.

We have found that contributions of various parameters to $\Delta m_{H_u}^2$ in first 6 orders of magnitude below M_{SUSY}^2 are very dense, with 13 parameters contributing, followed by a gap to below 10^{-10} level. As in the toy model, having many new parameters contributing in 6 orders of magnitude below M_{SUSY}^2 without gaps means that, specifying with one significant figure the way they are varied in small ranges around central values, the outcomes for the electroweak scale squared as small as $10^{-6} M_{SUSY}^2$, or $M_{EW} \simeq 10^{-3} M_{SUSY}$, occur with probabilities comparable to the most likely outcome.

As already discussed for the toy model, the results do depend on ranges of parameters and the way variations are specified. However, having 13 parameters contributing is significantly more than in the toy model and thus the conclusions would be comparable for large variations in the chosen procedure.

Discussion. Conclusions drawn in this letter are in a sharp contrast with commonly accepted views. In our analysis, the EW scale two orders of magnitude below superpartner masses, as suggested by the Higgs boson mass, is well within the range of comparably likely outcomes, $10^{-3} M_{SUSY} \lesssim M_{EW} \lesssim M_{SUSY}$, that do not need specifying any of the parameters with more than one significant figure. On the other hand, it is commonly argued that the EW scale resulting from 10 TeV superpartners, requires fine tuning of model parameters at the level of 1 part in 10^4 . This is usually estimated using Barbieri-Giudice measure, $|\partial \ln M_{EW}^2 / \partial \ln p_i|$, where p_i are model parameters and the role of M_{EW} is played by the Z boson mass, the Higgs mass or the vacuum expectation value in various versions [13][10].

This quantity, being a derivative, clearly represents the

sensitivity of the EW scale to model parameters. However, interpreting this sensitivity as the need for fine tuning or a sign of an unnatural outcome is misleading. In our toy model, the outcome $X \simeq 10^{-4}$ corresponds to $|\partial \ln X / \partial \ln A| \simeq 10^4$, possibly suggesting fine tuning needed at the level of 1 part in 10^4 just like for the EW symmetry breaking with 10 TeV superpartners. If there were just two model parameters, A and B , this would indeed be the level of fine tuning required: for any A , the B would have to be specified with 4 significant figures. However, as we saw, with more parameters contributing at smaller levels without significant gaps, none of the parameters needs to be specified with more than 1 significant figure.

Just like a blind application of the naturalness argument, this measure does not take into account the complexity of a given model and indicates the same for a model with two parameters as for a model with hundreds of parameters. The procedure followed in this letter, although somewhat flexible, automatically eliminates outcomes that would require very specific choices of parameters. In addition, it eliminates the need for a measure since it leads to a finite number of outcomes, probabilities of which can be calculated and compared.

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- [1] K. G. Wilson, Phys. Rev. D **3**, 1818 (1971).
- [2] G. 't Hooft, in *Recent developments in gauge theories*, Proceedings of the NATO Advanced Summer Institute, Cargese 1979, p. 135, Plenum Press, New York 1980.
- [3] E. Gildener, Phys. Rev. D **14**, 1667 (1976).
- [4] S. Weinberg, Phys. Lett. **82B**, 387 (1979).
- [5] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
- [6] M. J. G. Veltman, Acta Phys. Polon. B **12**, 437 (1981).
- [7] E. Witten, Nucl. Phys. B **188**, 513 (1981).
- [8] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24**, 1681 (1981).
- [9] For the first such a model, see S. Dimopoulos and H. Georgi, Nucl. Phys. B **193**, 150 (1981).
- [10] For reviews and references, see for example, S. P. Martin, Adv. Ser. Direct. High Energy Phys. **21**, 1 (2010) [hep-ph/9709356]; or, J. L. Feng, Ann. Rev. Nucl. Part. Sci. **63**, 351 (2013) [arXiv:1302.6587 [hep-ph]].
- [11] See, for example, P. Draper, G. Lee and C. E. M. Wagner, Phys. Rev. D **89**, no. 5, 055023 (2014) [arXiv:1312.5743 [hep-ph]].
- [12] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994) [hep-ph/9311340].
- [13] R. Barbieri and G. F. Giudice, Nucl. Phys. B **306** (1988) 63.